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A New Polynomial Channel Estimator of Multi Path Gains Based K-Means Algorithm for a Fast Time Varying MIMO OFDM System

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***Abstract*— In this paper, we propose a theoretical approach of a new estimator for a MIMO OFDM (Multi Input Multi Output Orthogonal Frequency Division Multiplexing) system which is based on the k- means algorithm. In fact, this approach is the first extension for the one proposed in a SISO OFDM.**

The variation of each complex gain is approximated using a polynomial form within various MIMO OFDM blocks.

We will show here the architecture of the proposed estimator as an analytical illustration and the simulations results will be further proposed in an extended version of this paper.

If we assume that the delay-related information is known, the polynomial coefficients are found from time-averaged gain values, which will be estimated basing on a least-square (LS) criterion. The channel matrix will be easily calculated, and the inter carriers interference ICI reduced via successive interference suppression through the data symbol detection. The algorithm's performance will be more enhanced by an iterative procedure, performing channel estimation and ICI improvement at each iteration.

We will give then an analysis of the mean square error of the proposed estimator and which will be further implemented in an extension version of this paper.

Theoretical analysis for a Rayleigh fading channel shows that the proposed algorithm has low computational complexity in the presence of high normalized Doppler spread.

***Index Terms*— MIMO OFDM, Channel Estimation, ICI, k-means, time varying, polynomial coefficients, LS.**

I. INTRODUCTION

In wireless communications, Multiple-Input-Multiple-Output (MIMO) systems with Orthogonal Frequency Division Multiplexing (OFDM) offer high data rates with a strong robustness to multi-path delay. In the one hand, the MIMO design represents a high-quality solution to attain both diversity and capacity gains [6]. In the other hand, OFDM is widely known as the promising communication technique in the recent broadband wireless mobile communication scheme thanks to its spectral efficiency and vigor to the multipath obstruction. Consequently, the combination of both MIMO and OFDM will allow a great challenge for next generation systems.

For diversity combining, coherent detection and decoding, channel parameters are required and the channel estimation has become crucial for MIMO-OFDM system design. In the literature, various OFDM channel estimation schemes have been proposed, typically for single antenna systems [1] [2] [3] [4] [5].

Most of the channel estimation schemes for MIMO OFDM systems have been studied for pilot- based approaches [7] [8] [9], which were adapted for a quasi-static fading or an invariant channel within a MIMO-OFDM block without forgotten the blind and semi blind approaches .

Nevertheless, , the time-variation of a fast fading channel in a MIMO-OFDM block outcome in a loss of sub- carriers' orthogonality which carry out the inter-carrier interference [1] [10]. To overcome the problem of the loss of orthogonality, we have to hold high speed mobile channels and their time-variation side within a block must be well considered.

Channel estimation can be kindly divided in two approaches.

The first one is based on the estimate of the equivalent discrete time channel taps [10] [2] whereas the second approach is to directly approximate the physical spread channel parameters like multipath delays and complex gains [11] [3] [1]. For Radio frequencies diffusion, the delays vary very slowly within several MIMO-OFDM blocks opposing to the complex gains which could change significantly even within one MIMO-OFDM block [12] [4].

Basing on the nature of the channel and supposing the availability of the delay information, several channel estimation methods of the multi-path complex gains were developed [7] [10] [4]. Note that there are more channel parameters for fast fading channels than those of the quasi static ones.

This paper is organized as follows. The second section illustrates the MIMO OFDM system model. In section III, we present multi path complex gains estimator and the iterative algorithm based k means. At last, in section IV, we will conclude this paper and we will give some prospects.

Notation - Column vectors (resp. matrices) are denoted with lower-case (resp. upper-case) bold letters; a_i (respectively $A_{i,j}$) denotes the i^{th} (respectively $(i,j)^{th}$) element of vector $\boldsymbol{a}$ (respectively matrix $\boldsymbol{A}$); diag($\boldsymbol{a}$) represents a diagonal matrix which has a main diagonal $\boldsymbol{a}$. $\boldsymbol{I}_N$ denotes the $N \times N$ identity matrix; $I_N^{(n)}$ refers to the n^{th} column of $\boldsymbol{I}_N$; e_N (resp. 0_N) signifies a vector of length N whose elements are 1 (resp. 0); E{.}, $(.)^*$, $(.)^T$ and $(.)^H$ stand for expectation, conjugate, transpose and conjugate transpose operators.

II. MIMO - OFDM SYSTEM MODEL

The MIMO OFDM system model considered consists of T_X transmitting antennas; R_X receiving antennas, N sub carriers, and the cyclic prefix is of length N_g. Denote the length of a MIMO-OFDM block as $T = \vartheta T_s$, wherever T_s is the sampling time with $\vartheta = \mathrm{N} + \mathrm{N_g}$. Let $x_{(n)} = \left[x_{(1,n)}^T, x_{(2,n)}^T, \dots, x_{(N_T,n)}^T\right]^T$ be the n^{th} transmitting MIMO OFDM block, $x_{(t,n)} = \left[x_{(t,n)}\left[-\frac{N}{2}\right], x_{(t,n)}\left[-\frac{N}{2}+1\right], \dots, x_{(t,n)}\left[\frac{N}{2}-1\right]\right]$ is the n^{th} transmitted OFDM symbol by the t^{th} transmit antenna and $\{x_{(t,n)}[b]\}$ are normalized symbols (i.e.,$E\left[x_{(t,n)}[b]x_{(t,n)}^*[b]\right] = 1$). Behind the transmission through the multipath Rayleigh channel, the n^{th} receiving MIMO OFDM block $r_{(n)} = \left[r_{(1,n)}^T, r_{(2,n)}^T, \dots, r_{(N_R,n)}^T\right]^T$ where $r_{(r,n)} = \left[r_{(r,n)}\left[-\frac{N}{2}\right], r_{(r,n)}\left[-\frac{N}{2}+1\right], \dots, r_{(r,n)}\left[\frac{N}{2}-1\right]\right]$ is the n^{th} received OFDM symbol by the r^{th} receiving antenna, and given by [4] [2]:

$$\mathbf{r}_{(n)} = \mathbf{H}_{(n)}\mathbf{x}_{(n)} + \mathbf{b}_{(n)} \tag{1}$$

Where $b_{(n)} = \left[b_{(1,n)}^T, b_{(2,n)}^T, \dots, b_{(N_R,n)}^T\right]^T$ and $b_{(r,n)} = \left[b_{(r,n)}\left[-\frac{N}{2}\right], b_{(r,n)}\left[-\frac{N}{2}+1\right], \dots, b_{(r,n)}\left[\frac{N}{2}-1\right]\right]$ is a white complex gaussian noise vector with a covariance matrix $N_T\sigma^2 I_N$ and $\mathbf{H}_{(n)}$ is a $N_T N \times N_R N$ MIMO channel matrix given by:

$$\mathbf{H}_{(n)} = \begin{bmatrix} \mathbf{H}_{(1,1,n)} & \cdots & \mathbf{H}_{(1,N_T,n)} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{(1,N_R,n)} & \cdots & \mathbf{H}_{(N_R,N_T,n)} \end{bmatrix} \tag{2}$$

Wherever $\mathbf{H}_{(r,t,n)}$ represents the channel matrix among the t^{th} transmitting antenna and the r^{th} receiving one.

Note that the delays are normalized by means of T_s and are not essentially integers $T_{(l,r,t)} < N_\mathrm{g}$.

III. MULTI - PATH GAINS ESTIMATOR BASED K-MEANS ALGORITHM

Assume that the proposed method is based on comb- type pilots' arrangements and multi- path time delays information for estimating the sampled complex gains $\alpha_l(qT_s)$ with T_s denoting for the sampling period.

The N_p pilot subcarriers will be regularly introduced into the N subcarriers related to a comb-type scheme. The interval between two adjacent pilots in terms of number of subcarriers in the frequency domain is denoted as L_f which can be chosen without require for respect the sampling theorem.

Nevertheless, N_p should perform the following constraint: $N_p \geq L$ (L denotes the number of paths).

Moreover, P represents the set containing the index positions of the N_p pilot sub-carriers and is expressed as:

$$P = \left\{p_s / p_s = sL_f - \frac{N}{2}, s = 0, \dots, N_p - 1\right\} \tag{3}$$

The received pilot subcarriers from the i^{th} transmit antenna and the j^{th} receive antenna can be written as:

$$R_p^{(i,j)} = \underline{\underline{X_p^{(i,j)}}}\,\underline{H_p}\,\underline{\underline{H_{PI}}}\,\underline{X} + \underline{B_p} \tag{4}$$

Where $\underline{\underline{X_p^{(i,j)}}}$ is an $N_p \times N_p$ diagonal matrix, $R_p^{(i,j)}$ and $\underline{B_p}$ are $N_p \times 1$ vectors given by:

$$\underline{\underline{X_p^{(i,j)}}} = diag\left\{X[p_0]^{(i,j)}, X[p_1]^{(i,j)}, \dots, X[p_{N_p-1}]^{(i,j)}\right\}$$

$$R_p^{(i,j)} = \left\{R[p_0]^{(i,j)}, R[p_1]^{(i,j)}, \dots, R[p_{N_p-1}]^{(i,j)}\right\}^T$$

$$\underline{B_p} = \left\{B[p_0]^{(i,j)}, B[p_1]^{(i,j)}, \dots, B[p_{N_p-1}]^{(i,j)}\right\}^T$$

$\underline{H_p}$ is an $N_p \times 1$ vector and $\underline{\underline{H_{PI}}}$ is an $N_p \times N$ matrix with elements expressed as:

$$H_p[p_s] = H[p_s, p_s] = \sum_{l=1}^{L} (\overline{\alpha_l})^{(i,j)}\, e^{-j2\pi \frac{p_s}{N}\tau_l}$$

$$H_{pI}[p_s, m] = \begin{cases} H[p_s, m]\ if\ \ m \in \left[-\frac{N}{2}, \frac{N}{2} - 1\right] - P \\ \quad 0\ \ if\ m \in P \end{cases}$$

$$\text{with} \qquad (\overline{\alpha_l})^{(i,j)} = \left[\frac{1}{N}\sum_{q=0}^{N-1} \alpha_l(qT_s)\right]^{(i,j)} \tag{5}$$

$(\overline{\alpha_l})^{(i,j)}$ is the time average over the effective duration of the MIMO OFDM block of the l^{th} complex gain through the i^{th} transmitting antenna and the j^{th} receiving antenna. The first factor is the wanted term without inter carrier interference and the second part is the inter carriers interference expression.

$\underline{H_p}$ represents the Fourier transform for the different complex gains time average $\{(\overline{\alpha_l})^{(i,j)}\}$:

$$\underline{H_p} = \underline{\underline{F_P}}\, (\underline{\bar{\alpha}})^{(i,j)} \tag{6}$$

Where $\underline{\underline{F_P}}$ and $(\underline{\bar{\alpha}})^{(i,j)}$ are respectively the $N_p \times L$ Fourier transform matrix and the vector

$$\underline{\underline{F_P}} = \begin{pmatrix} e^{-j2\pi\frac{p_0}{N}\tau_1} & \cdots & e^{-j2\pi\frac{p_0}{N}\tau_L} \\ \vdots & \ddots & \vdots \\ e^{-j2\pi\frac{p_{N_p-1}}{N}\tau_1} & \cdots & e^{-j2\pi\frac{p_{N_p-1}}{N}\tau_L} \end{pmatrix} \tag{7}$$

$$(\underline{\bar{\alpha}})^{(i,j)} = \left[(\overline{\alpha_1})^{(i,j)}, (\overline{\alpha_2})^{(i,j)}, \dots, (\overline{\alpha_L})^{(i,j)}\right]^T$$

For the different paths, the complex gains time averages above the effective interval of each MIMO OFDM block, are estimated, through the i^{th} transmit antenna and j^{th} receive antenna, by means of the LS criterion. Via neglecting the inter carriers interference contribution, the LS estimator of the gain $\alpha^{(i,j)}$ is:

$$(\underline{\bar{\alpha}})^{(i,j)}{}_{LS} = MR_p^{(i,j)} \tag{8}$$

With $\quad M = \left(F_p^H \left(X_p^{(i,j)}\right)^H X_p^{(i,j)} F_p\right)^{-1} F_p^H (X_p^{(i,j)})^H$

Because the T_s spaced sampling of the complex gain, token in the middle of the effective duration of a MIMO OFDM block, is nearby to the complex gain time average over the effective duration, then we suppose that $(\underline{\bar{\alpha}})^{(i,j)}{}_{LS}$ is an estimation of $\underline{\alpha_c}^{(i,j)} = \left[\alpha_1^{(i,j)}(\frac{N}{2}T_s), \dots, \alpha_L^{(i,j)}(\frac{N}{2}T_s)\right]^T$.

So, by estimating complex gains for some MIMO OFDM blocks and interpolating them by the factor $(N + N_g)$ by using low-pass interpolation [12], we obtain, for each path, an estimation of the sampled complex gains $\{\alpha_l(qT_s)\}$ at moment T_s during these MIMO OFDM blocks.

- *Algorithm of the proposed estimator:*

For the i^{th} transmitting and j^{th} receiving antennas and in the iterative algorithm of channel estimation and inter carriers interference suppression, the MIMO OFDM blocks are grouped in blocks of K MIMO OFDM blocks each one. Each two consecutive blocks are intersected in two MIMO OFDM blocks. For a block of K MIMO OFDM blocks, the iterative algorithm proceeds as following:

Initialization of the algorithm:

$m \leftarrow 1$

$\underline{R_p^{(k,m)}}_{(i,j)} = \underline{R_p^k}_{(i,j)}$

Recursion:

1. $(\underline{\bar{\alpha}})^{(k,m)}{}_{LS\,(i,j)} = M\underline{R_p^{k,m}}_{(i,j)}$
2. $\left\{ \begin{matrix} (\hat{\alpha}_l^{k,m}(qT_s))_{(i,j)}, k = 2, \dots, K-1; \\ q = -N_g, \dots, N-1 \end{matrix} \right\} = interp(\overline{\alpha}_l^{k,m}{}_{LS}, N + N_g)$
3. Compute the channel matrix using the $\underline{\underline{\hat{H}}}^{k,m}$ coefficients of the channel matrix from the n^{th} transmitted subcarrier frequency to the r^{th} received subcarrier frequency.
4. Remove the inter carriers interference from the received data subcarriers $\underline{R_d^k}_{(i,j)}$
5. Detection of the data symbols $\underline{\hat{X}_d}^{k,m}_{(i,j)}$
6. $\underline{R_p^{k,m+1}}_{(i,j)} = \underline{R_p^k}_{(i,j)} - \underline{\underline{\widehat{H_{PI}}}}^{k,m} \underline{\hat{X}}^{k,m}{}_{(i,j)}$
7. $m \leftarrow m + 1$

With *interp* is a Matlab the interpolation function and, m and k represent, respectively, the iteration number and the number of a MIMO OFDM symbol in a block. Note that, the steps 3 to 6 are accomplished without taking into account the first and the last MIMO OFDM blocks (ie. K=1 to K-1) in order to pass up limiting effects of interpolation. The detected data symbols, in the fifth step, are estimated by the SIS scheme (successive data interference suppression) with the optimal classification and one tap frequency equalizer [13] [14].

- Analysis of the Mean Square Error:

The performance of each estimator should be analysed with an important parameter. In our case, we choose the mean square error (MSE). Then, for the i^{th} transmit antenna and j^{th} receive antenna, the MSE of the least square estimator LS of the multi path complex gain is given by:

$$MSE = E[(\underline{\bar{\alpha}})^{(i,j)}{}_{LS} - (\underline{\bar{\alpha}})^{(i,j)})] \quad (9)$$

$$= Tr(MTM^H)$$

With $T = E\left[\underline{\underline{H_{PI}}}\,\underline{X}\ \underline{X}^H\underline{\underline{H_{PI}}}^H\right] + \sigma^2\underline{\underline{I}}_{N_p}$

So, for the evaluation of the performance of the LS estimator, we can resort to the Cramer - Rao Bound (CRB) which provides the MMSE bound for unbiased estimators since that our estimator seems to be unbiased from its analytical form.

Hence, the Standard Cramer Rao Bound (SCRB) for the estimation of the multi path complex gains with a known inter carriers' interference ICI can be expressed as:

$$SCRB((\underline{\bar{\alpha}})^{(i,j)}{}_{LS}) = \frac{1}{SNR} Tr\left(\left(F_p^H \left(X_p^{(i,j)}\right)^H X_p^{(i,j)} F_p\right)^{-1}\right) \quad (10)$$

With SNR denotes the normalized Signal to Noise Ratio. From what the inter carriers' interference ICI will be suppressed or not, it will be easy then to demonstrate that the MSE will be grater or not than the standard CRB.

Taking into account of all these consequences, we can see that in the presence of the ICI, the MSE will be grater than SCRB of the multipath gains, else and without ICI it will be equal to this bound.
Consequently, via estimating and eliminating, the ICI, iteratively, the MSE will be closer to $SCRB((\underline{\bar{\alpha}})^{(i,j)}{}_{LS})$.

Then, for the i^{th} transmit antenna and j^{th} receive antenna, the MSE of the supposition that the LS estimator of the multi path gain is an estimation of this gain at the effective duration is expressed as:

$$MSE = E[((\underline{\bar{\alpha}})^{(i,j)}{}_{LS} - \underline{\alpha_c}^{(i,j)})^H \left((\underline{\bar{\alpha}})^{(i,j)}{}_{LS} - \underline{\alpha_c}^{(i,j)}\right) \quad (11)$$

In fact, this expression consists of several terms which concerns in the one hand the MSE between $(\underline{\bar{\alpha}})^{(i,j)}$ and $\underline{\alpha_c}^{(i,j)}$ and the cross covariance terms between them in the other hand. We should note also that the later terms are very negligible

Finally, for the i^{th} transmit antenna and j^{th} receive antenna, the Mean Square Error at the sampling time T_s of the multi path gain estimator is expressed as:

$$MSE_{T_s}(i,j)) = \sum_{k=2}^{K-1}\sum_{q=-N_g}^{N-1} E\left[\left(\left(\underline{\hat{\alpha}}_q^k - \underline{\alpha}_q^k\right)^H \left(\underline{\hat{\alpha}}_q^k - \underline{\alpha}_q^k\right)\right)^{(i,j)}\right]$$

$$\text{With } \underline{\alpha}_q^k = [\alpha_1^k(qT_s), \dots, \alpha_L^k(qT_s)]^T \quad (12)$$

If we assume that we have a performing interpolation scheme and with respect to the sampling theorem in the temporal domain, we have a quasi equality between the MSE at time T_s and which at the effective duration.

IV. CONCLUSION

In this paper, we had proposed a new iterative algorithm for estimating multipath complex gains and reducing the inter sub- carrier-interference (ICI) for a MIMO OFDM time varying system. Since that our proposition is build for a MIMO OFDM context and is an extension for a SISO OFDM case, we hope that the performances of our estimator will track the former case.
In fact, the theoretical analysis and simulation results of the iterative algorithm for an OFDM system show that, by estimating and eliminating the inter carriers interference at each iteration, multipath complex gains estimation and coherent demodulation had a big enhancement especially for high Doppler spread.

As we had mentioned beyond, we are currently preparing the simulations results of this work which will be drawn in an extended edition of this paper.

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